

Revisit on partial solutions in the Adomian decomposition method: Solving heat and wave equations[☆]

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Abstract

The Adomian decomposition method is considered in application to heat and wave equations. The so-called partial solution technique is used. It is shown that the fundamental equation of the method is well defined only for certain types of boundary conditions. In cases involving inhomogeneous boundary conditions, improper results may be obtained by former method. This paper presents a further insight into partial solutions in the decomposition method, and the resolution of such cases.

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1. Introduction

The Adomian decomposition method (ADM) is a creative and effective method for exactly solving functional equations of various kinds. It is important to note that a large amount of research work has been devoted to the application of the ADM to a wide class of linear and non-linear, ordinary or partial differential equations [1–27]. The decomposition method provides the

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solution as an infinite series in which each term can be easily determined. The rapid convergence of the series obtained by this method is thoroughly discussed by Cherruault et al. in [23] and the references therein.

As we all know, it was initially established by Adomian that in partial differential equation problems involving linear operator terms with respect to x, y, z, t , four equations are solved and then a linear combination of these solutions is necessary [1–6]. It was found recently by Adomian and Rach [9] that in partial differential equations, considerable work is saved by solving only one of the linear operator equations to obtain a so-called partial solution. However, in cases where one operator annihilates the series in a finite number of terms, one partial solution may not satisfy the given conditions. In addition, in cases involving inhomogeneous boundary conditions, the result obtained by the partial solution technique may be improper.

In this paper, we present a further insight into partial solutions in the decomposition method, and the resolution of above cases by suitable transformation. For completeness we first mention some of the early studies on partial solutions in the Adomian decomposition method. These studies were summarized and discussed in great detail in [9,11].

2. Review of partial solutions

Recently, Adomian and Rach [9] presented a thorough investigation of equality of partial solutions in the decomposition method for linear or nonlinear partial differential equations. They concluded that the partial solutions obtained from the separate equations for the highest-ordered linear operator terms were shown to be identical when the boundary conditions are general, and asymptotically equal when the boundary conditions in one independent variable are independent of other variables. It is important to note that the solution to either equations are equal except for special cases discussed in [9,10].

More recently, Adomian and Rach [11] provided a further insight into cases where one operator annihilates the series in a finite number of terms, one partial solution may not satisfy the given conditions. The resolution of such cases was obtained by appropriate expansion of the initial term without making a priori assumptions about the solution. However, as will be seen from the examples below, this technique is failed to obtain exact solution in the case of initial-boundary value problems with inhomogeneous boundary conditions.

Exactly the same problem was seemed to be initially considered by Adomian in [7]. Consider the heat equation

$$u_t = u_{xx}, \quad u(0, t) = t, \quad u_x(0, t) = 0, \quad u(x, 0) = \frac{x^2}{2}. \quad (1)$$

It was suggested by Adomian that it must be solved in the standard way, using two equations for u —one inverting the L_t operator and one inverting the L_x operator—adding, and dividing by two. However, as for the same equation with the conditions $u(0, t) = u(\pi, t) = 0$, $u(x, 0) = \sin x$ only $L_t u$ is solved and the exact solution $u = e^{-t} \sin x$ can be obtained without the slow convergence phenomenon in [4].

The present work is mainly motivated by the previous works with the objective to offer exact solutions for applications. A review of previous studies indicates that the difficulties arise when trying to apply the Adomian decomposition method to partial differential equations with inhomogeneous boundary conditions.

3. The heat equation

3.1. Homogeneous heat equation with homogeneous boundary conditions

We consider a homogeneous rod of length ℓ . The rod is sufficiently thin so that the heat is distributed equally over the cross section at time t . The surface of the rod is insulated, and therefore there is no heat loss through the boundary. The temperature distribution of the rod is given by the solution of the initial-boundary value problem

$$\begin{aligned} u_t &= u_{xx}, & 0 < x < \ell, \quad t > 0, \\ u(0, t) &= 0, & t \geq 0, \\ u(\ell, t) &= 0, & t \geq 0, \\ u(x, 0) &= f(x), & 0 \leq x \leq \ell. \end{aligned} \quad (2)$$

In cases where $f(x)$ is “well-defined,” namely the partial solution technique is used and the result which satisfy the boundary conditions is obtained, modification is unnecessary. For example, we choose $f(x) = \sin(\pi x/\ell)$ and the t equation in the usual notation of the reference work is

$$L_t u = L_x u, \quad (3)$$

where $L_t = \partial/\partial t$, $L_x = \partial^2/\partial x^2$. Applying the operator $L_t^{-1}(\cdot) = \int_0^t (\cdot) dt$, we get [9]

$$\begin{aligned} u &= u(x, 0) + L_t^{-1} L_x \sum_{n=0}^{\infty} u_n, \\ u_0 &= f(x) = \sin(\pi x/\ell), \\ u_1 &= L_t^{-1} L_x u_0 = -(\pi^2 t/\ell^2) \sin(\pi x/\ell), \\ u_2 &= L_t^{-1} L_x u_1 = (\pi^4 t^2/\ell^4) \sin(\pi x/\ell), \\ &\vdots \\ u &= \sum_{n=0}^{\infty} u_n = (1 - (\pi^2 t)/\ell^2 + (\pi^4 t^2)/\ell^4 - \dots) \sin(\pi x/\ell) \\ &= e^{-\pi^2 t/\ell^2} \sin(\pi x/\ell). \end{aligned} \quad (4)$$

In cases where $f(x)$ is “ill-defined,” namely the L_t^m operator acting on $f(x)$ annihilates the series in a finite number of terms and the result does not satisfy the boundary conditions, we expand $f(x)$ in an infinite series to prevent the annihilation. For example, we consider the same boundary conditions but choose $f(x) = x(\ell - x)$ which suggested by Professor Y. Cherrault [11]. Thus,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right), \quad b_n = \frac{2}{\ell} \int_0^{\ell} f(x) \sin\left(\frac{n\pi x}{\ell}\right) dx. \quad (5)$$

Now

$$u = \sum_{m=0}^{\infty} [L_t^{-1} L_x]^m f(x)$$

$$\begin{aligned}
&= \sum_{m=0}^{\infty} [L_t^{-1} L_x]^m \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right) \\
&= \sum_{m=0}^{\infty} \frac{t^m}{m!} \frac{\partial^{2m}}{\partial x^{2m}} \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right) \\
&= \sum_{m=0}^{\infty} (-1)^m \frac{t^m}{m!} \sum_{n=1}^{\infty} \left(\frac{n\pi}{\ell}\right)^{2m} b_n \sin\left(\frac{n\pi x}{\ell}\right) \\
&= \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right) \sum_{m=0}^{\infty} (-1)^m \left(\frac{n\pi}{\ell}\right)^{2m} \frac{t^m}{m!} \\
&= \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right) e^{-n^2 \pi^2 t / \ell^2}, \tag{6}
\end{aligned}$$

which is the Fourier expansion of the solution $u(x, t)$ derived by rearrangement of the decomposition solution of the t equation.

3.2. Homogeneous heat equation with inhomogeneous boundary conditions

Now let us consider the same equation with inhomogeneous boundary conditions $u(0, t) = T_1$, $u(\ell, t) = T_2$, and initial condition $u(x, 0) = h(x)$ [11]. T_1, T_2 are numerical constants. $h(x) \rightarrow T_1$ when $x \rightarrow 0$, and $h(x) \rightarrow T_2$ when $x \rightarrow \ell$. The result obtained in [11] was

$$u = \sum_{n=1}^{\infty} \Psi_n \sin\left(\frac{n\pi x}{\ell}\right) e^{-n^2 \pi^2 t / \ell^2}, \quad \Psi_n = \frac{2}{\ell} \int_0^{\ell} h(x) \sin\left(\frac{n\pi x}{\ell}\right) dx. \tag{7}$$

The “steady-state” solution $u = T_1 + (T_2 - T_1)x/\ell$ was also obtained in [11]. In addition, it was suggested that if the T_1, T_2 are functions instead of numerical constants, they can be expanded in Fourier terms to get a complete solution. For more details, readers are referred to [11].

However, in view of (7) and inhomogeneous boundary conditions $u(0, t) = T_1$, $u(\ell, t) = T_2$, it is clear that the result (7) does not satisfy the boundary conditions. Unfortunately, there was no consideration about this wrong result in [11]. In fact, we can deal with this difficulty with the help of transformation introduced in [28,29]. For convenience, we rewrite the problem

$$\begin{aligned}
u_t &= u_{xx}, \quad 0 < x < \ell, \quad t > 0, \\
u(0, t) &= T_1, \quad t \geq 0, \\
u(\ell, t) &= T_2, \quad t \geq 0, \\
u(x, 0) &= h(x), \quad 0 \leq x \leq \ell. \tag{8}
\end{aligned}$$

Let

$$u(x, t) = v(x, t) + \omega(x), \quad f(x) = h(x) - \omega(x), \tag{9}$$

where

$$\omega(x) = T_1 + \frac{T_2 - T_1}{\ell} x. \tag{10}$$

Substitution of $u(x, t)$ in problem (8) yields

$$\begin{aligned} v_t &= v_{xx}, \quad 0 < x < \ell, \quad t > 0, \\ v(0, t) &= 0, \quad t \geq 0, \\ v(\ell, t) &= 0, \quad t \geq 0, \\ v(x, 0) &= f(x), \quad 0 \leq x \leq \ell. \end{aligned} \quad (11)$$

Hence with the knowledge of solution (6), we obtain the solution

$$u(x, t) = v(x, t) + \omega(x) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{\ell}\right) e^{-n^2\pi^2 t/\ell^2} + T_1 + \frac{T_2 - T_1}{\ell}x, \quad (12)$$

where

$$c_n = \frac{2}{\ell} \int_0^{\ell} (f(\tau) - \omega(\tau)) \sin \frac{n\pi \tau}{\ell} d\tau, \quad (13)$$

which satisfies inhomogeneous boundary conditions.

3.3. Inhomogeneous heat equation with homogeneous boundary conditions

Consider the inhomogeneous case which is not solvable by separation of variables

$$L_t u = L_x u + h(x, t), \quad (14)$$

and choose $u(x, 0) = u(0, t) = u(\ell, t) = 0$ [11]. Solving for $L_t u$, we have

$$\begin{aligned} u_0 &= L_t^{-1} h + u(x, 0) = L_t^{-1} h, \\ u_m &= (L_t^{-1} L_x)^m u_0, \quad m \geq 0, \\ u &= \sum_{m=0}^{\infty} u_m. \end{aligned} \quad (15)$$

In cases where $h(x, t)$ is “well-defined,” additional consideration is unnecessary. For example, assume $h(x, t) = \ell x - x^2 + 2t$, we have

$$\begin{aligned} u_0 &= L_t^{-1} h = \int_0^t (\ell x - x^2 + 2t) dt = (\ell x - x^2)t + t^2, \\ u_1 &= L_t^{-1} L_x u_0 = -t^2, \\ u_2 &= L_t^{-1} L_x u_1 = 0, \\ u &= \sum_{m=0}^{\infty} u_m = u_0 + u_1 = (\ell x - x^2)t, \end{aligned} \quad (16)$$

which is the exact solution.

In cases where $h(x, t)$ is “ill-defined,” namely the L_x^m operator acting on h annihilates the series in a finite number of terms and the result does not satisfy the boundary conditions, we consider the Fourier expansion of h [11]. Thus, the solution is

$$u(x, t) = \sum_{n=1}^{\infty} \delta_n(t) \sin\left(\frac{n\pi x}{\ell}\right), \quad (17)$$

where

$$\begin{aligned} \delta_n(t) &= \sum_{m=0}^{\infty} (-1)^m \left(\frac{n\pi x}{\ell}\right)^{2m} L_t^{-(m+1)} \gamma_n(t), \\ \gamma_n(t) &= \frac{2}{\ell} \int_0^{\ell} h(x, t) \sin\left(\frac{n\pi x}{\ell}\right) dx, \\ L_t^{-1}(\cdot) &= \int_0^t (\cdot) dt. \end{aligned} \quad (18)$$

3.4. Inhomogeneous heat equation with inhomogeneous boundary conditions

As seen in Section 3.2, the result obtained in [11] does not satisfy the inhomogeneous boundary conditions. Hence, it is important to consider the inhomogeneous heat equation with inhomogeneous boundary conditions. Let us consider the problem

$$\begin{aligned} u_t &= u_{xx} + h(x, t), \quad 0 < x < \ell, \quad t > 0, \\ u(0, t) &= p(t), \quad t \geq 0, \\ u(\ell, t) &= q(t), \quad t \geq 0, \\ u(x, 0) &= f(x), \quad 0 \leq x \leq \ell. \end{aligned} \quad (19)$$

In a similar manner, let

$$u(x, t) = v(x, t) + \omega(x, t), \quad (20)$$

where

$$\omega(x, t) = \frac{\ell - x}{\ell} p(t) + \frac{x}{\ell} q(t). \quad (21)$$

Substitution of $u(x, t)$ in problem (19) yields

$$\begin{aligned} v_t &= v_{xx} + H(x, t), \quad 0 < x < \ell, \quad t > 0, \\ v(0, t) &= 0, \quad t \geq 0, \\ v(\ell, t) &= 0, \quad t \geq 0, \\ v(x, 0) &= \Psi(x), \quad 0 \leq x \leq \ell, \end{aligned} \quad (22)$$

where

$$H(x, t) = h(x, t) - \omega_t(x, t), \quad \Psi(x) = f(x) - \omega(x, 0). \quad (23)$$

Solving for $L_t v$, we have

$$\begin{aligned}
v_0 &= \Psi + L_t^{-1} H = \varphi(x, t), \\
v_m &= (L_t^{-1} L_x)^m v_0, \quad m \geq 0, \\
v &= \sum_{m=0}^{\infty} v_m.
\end{aligned} \tag{24}$$

In cases where $\varphi(x, t)$ is “ill-defined,” we consider the Fourier expansion of φ . Thus, v can be found as above and (20) yields the solution u to the corresponding inhomogeneous problem.

4. The wave equation

4.1. Homogeneous wave equation with homogeneous boundary conditions

We consider the problem of the vibrating string stretched along the x -axis from 0 to ℓ , fixed at its end points. The problem is given by

$$\begin{aligned}
L_t u &= L_x u, \quad 0 < x < \ell, \quad t > 0, \\
u(0, t) &= 0, \quad t \geq 0, \\
u(\ell, t) &= 0, \quad t \geq 0, \\
u(x, 0) &= f(x), \quad 0 \leq x \leq \ell, \\
u_t(x, 0) &= g(x), \quad 0 \leq x \leq \ell,
\end{aligned} \tag{25}$$

where $L_t = \partial^2 / \partial t^2$, $L_x = \partial^2 / \partial x^2$.

In cases where conditions are “well-defined,” the exact solution is easily obtained. As a simple example for clarity, assume that $\ell = \pi$ and initial conditions $u(x, 0) = f(x) = 0$ and $u_t(x, 0) = g(x) = \sin x$. Following the partial solution technique, and applying the operator $L_t^{-1}(\cdot) = \int_0^t \int_0^t (\cdot) dt dt$ on both sides of (25), we have

$$\begin{aligned}
u &= u(x, 0) + t u_t(x, 0) + L_t^{-1} L_x \sum_{n=0}^{\infty} u_n, \\
u_0 &= t \sin x, \\
u_1 &= L_t^{-1} L_x u_0 = -\frac{t^3}{3!} \sin x, \\
u_2 &= L_t^{-1} L_x u_1 = \frac{t^5}{5!} \sin x, \\
&\vdots \\
u &= \sum_{n=0}^{\infty} u_n = \left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \cdots \right) \sin x = \sin t \sin x,
\end{aligned} \tag{26}$$

which is the exact solution.

In cases where conditions are “ill-defined,” we can expand $f(x)$ and $g(x)$ in Fourier terms and the exact solution is [11]

$$u(x, t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{\ell}\right) \left\{ a_n \cos\left(\frac{n\pi t}{\ell}\right) + b_n \frac{\sin(n\pi t/\ell)}{(n\pi/\ell)} \right\}, \tag{27}$$

where

$$a_n = \frac{2}{\ell} \int_0^{\ell} f(x) \sin\left(\frac{n\pi x}{\ell}\right) dx, \quad b_n = \frac{2}{\ell} \int_0^{\ell} g(x) \sin\left(\frac{n\pi x}{\ell}\right) dx. \quad (28)$$

4.2. Homogeneous wave equation with inhomogeneous boundary conditions

As seen before, the difficulties arise when trying to apply the method proposed in [11] to partial differential equations with inhomogeneous boundary conditions. It is necessary to consider the homogeneous wave equation with inhomogeneous boundary conditions

$$\begin{aligned} L_t u &= L_x u, & 0 < x < \ell, \quad t > 0, \\ u(0, t) &= A, & t \geq 0, \\ u(\ell, t) &= B, & t \geq 0, \\ u(x, 0) &= h(x), & 0 \leq x \leq \ell, \\ u_t(x, 0) &= g(x), & 0 \leq x \leq \ell. \end{aligned} \quad (29)$$

In a similar manner, let

$$u(x, t) = v(x, t) + \omega(x), \quad f(x) = h(x) - \omega(x), \quad (30)$$

where

$$\omega(x) = A + \frac{B - A}{\ell} x. \quad (31)$$

Substitution of $u(x, t)$ in problem (29) yields

$$\begin{aligned} L_t v &= L_x v, & 0 < x < \ell, \quad t > 0, \\ v(0, t) &= 0, & t \geq 0, \\ v(\ell, t) &= 0, & t \geq 0, \\ v(x, 0) &= f(x), & 0 \leq x \leq \ell, \\ u_t(x, 0) &= g(x), & 0 \leq x \leq \ell. \end{aligned} \quad (32)$$

Hence with the knowledge of last section and (30), the solution u to the corresponding inhomogeneous problem can be obtained.

4.3. Inhomogeneous wave equation with homogeneous boundary conditions

In practice, there is a very important class of problems involving inhomogeneous equations. Let us consider the problem

$$\begin{aligned} L_t u &= L_x u + h(x, t), & 0 < x < \ell, \quad t > 0, \\ u(0, t) &= 0, & t \geq 0, \\ u(\ell, t) &= 0, & t \geq 0, \\ u(x, 0) &= f(x), & 0 \leq x \leq \ell, \\ u_t(x, 0) &= g(x), & 0 \leq x \leq \ell. \end{aligned} \quad (33)$$

In cases where conditions are “well-defined,” the exact solution can be obtained by using usual Adomian decomposition method. However, in cases where conditions are “ill-defined,” we consider the Fourier expansion of $L_t^{-1}h$, f and g , thus

$$\begin{aligned} H(x, t) &= \sum_{n=1}^{\infty} a_n(t) \sin\left(\frac{n\pi x}{\ell}\right), \quad a_n(t) = \frac{2}{\ell} \int_0^{\ell} H(x, t) \sin\left(\frac{n\pi x}{\ell}\right) dx, \\ f(x) &= \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right), \quad b_n = \frac{2}{\ell} \int_0^{\ell} f(x) \sin\left(\frac{n\pi x}{\ell}\right) dx, \\ g(x) &= \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{\ell}\right), \quad c_n = \frac{2}{\ell} \int_0^{\ell} g(x) \sin\left(\frac{n\pi x}{\ell}\right) dx, \end{aligned} \quad (34)$$

where $H(x, t) = L_t^{-1}h(x, t)$. As a simple example for clarity, we will study the problem taken from [28, Example 6.7.2, p. 155], assume that $\ell = 1$, $h(x, t) = h = \text{constant}$ and initial conditions $u(x, 0) = f(x) = x(1 - x)$ and $u_t(x, 0) = g(x) = 0$. Thus,

$$\begin{aligned} H(t) &= \frac{h}{2}t^2 = \sum_{n=1}^{\infty} a_n(t) \sin(n\pi x), \quad a_n(t) = \frac{ht^2}{n\pi} [1 - (-1)^n], \\ f(x) &= \sum_{n=1}^{\infty} b_n \sin(n\pi x), \quad b_n = \frac{4}{(n\pi)^3} [1 - (-1)^n], \end{aligned} \quad (35)$$

so that

$$\begin{aligned} u &= u(x, 0) + tu_t(x, 0) + L_t^{-1}h(x, t) + L_t^{-1}L_x \sum_{n=0}^{\infty} u_n, \\ u_0 &= f(x) + H(t) = \sum_{n=1}^{\infty} b_n \sin(n\pi x) + \sum_{n=1}^{\infty} a_n(t) \sin(n\pi x), \\ u_m &= (-1)^m \frac{t^{2m}}{(2m)!} \sum_{n=1}^{\infty} b_n (n\pi)^{2m} \sin(n\pi x) + (-1)^m \frac{t^{2m+2}}{(2m+2)!} \sum_{n=1}^{\infty} c_n (n\pi)^{2m} \sin(n\pi x), \\ u &= \sum_{m=0}^{\infty} u_m, \\ u &= \sum_{n=1}^{\infty} b_n \sin(n\pi x) \sum_{m=0}^{\infty} (-1)^m \frac{t^{2m}}{(2m)!} (n\pi)^{2m} \\ &\quad + \sum_{n=1}^{\infty} c_n \sin(n\pi x) \sum_{m=0}^{\infty} (-1)^m \frac{t^{2m+2}}{(2m+2)!} (n\pi)^{2m} \\ &= \sum_{n=1}^{\infty} \sin(n\pi x) \left\{ b_n \cos(n\pi t) + c_n \frac{1 - \cos(n\pi t)}{(n\pi)^2} \right\}, \end{aligned} \quad (36)$$

where

$$c_n = \frac{2h}{n\pi} [1 - (-1)^n]. \quad (37)$$

It is important to note that the result (36) is also obtained in [28] by different method.

4.4. Inhomogeneous wave equation with inhomogeneous boundary conditions

We will now consider the initial-boundary value problem with the time-dependent boundary conditions, namely,

$$\begin{aligned} L_t u &= L_x u + h(x, t), \quad 0 < x < \ell, \quad t > 0, \\ u(0, t) &= p(t), \quad t \geq 0, \\ u(\ell, t) &= q(t), \quad t \geq 0, \\ u(x, 0) &= f(x), \quad 0 \leq x \leq \ell, \\ u_t(x, 0) &= g(x), \quad 0 \leq x \leq \ell. \end{aligned} \quad (38)$$

Let

$$u(x, t) = v(x, t) + \omega(x, t), \quad (39)$$

where

$$\omega(x, t) = \frac{\ell - x}{\ell} p(t) + \frac{x}{\ell} q(t). \quad (40)$$

Substitution of $u(x, t)$ in problem (38) yields

$$\begin{aligned} L_t v &= L_x v + H(x, t), \quad 0 < x < \ell, \quad t > 0, \\ v(0, t) &= 0, \quad t \geq 0, \\ v(\ell, t) &= 0, \quad t \geq 0, \\ v(x, 0) &= F(x), \quad 0 \leq x \leq \ell, \\ v_t(x, 0) &= G(x), \quad 0 \leq x \leq \ell, \end{aligned} \quad (41)$$

where

$$H(x, t) = h - \omega_{tt}, \quad F(x) = f(x) - \omega(x, 0), \quad G(x) = g(x) - \omega_t(x, 0). \quad (42)$$

Hence with the knowledge of last section, the solution can be obtained.

5. Discussions

The heat and wave equations are solved by so-called partial solution technique. In cases where boundary conditions are “well-defined,” the solution can be obtained by the usual decomposition method. In cases where boundary conditions are “ill-defined,” appropriate expansion of the initial term should be used as in [11]. However, improper solution may be obtained when consider the initial boundary value problems involving inhomogeneous boundary conditions, and there are many such problems can be found in [28,29]. We overcome the difficulties with the help of suitable transformation and the same results as in [28] are successfully obtained. However, it should be noted that the transformation used in this paper is a simple technique for special cases and the powerful Adomian decomposition method can also be applied to much more complicated physical problems in comparison with other methods.

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